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Field and Wave Electromagnetics

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Preface

The many books on introductory electromagnetics can be roughly divided into two main groups. The first group takes the traditional development: starting with the experimental laws, generalizing them in steps, and finally synthesizing them in the form of Maxwell's equations. This is an inductive approach. The second group takes the axiomatic development: starting with Maxwell's equations, identifying each with the appropriate experimental law, and specializing the general equations to static and time-varying situations for analysis. This is a deductive approach. A few books begin with a treatment of the special theory of relativity and develop all of electromagnetic theory from Coulomb's law of force; but this approach requires the discussion and understanding of the special theory of relativity first and is perhaps best suited for a course at an advanced level.

Proponents of the traditional development argue that it is the way electromagnetic theory was unraveled historically (from special experimental laws to Maxwell's equations), and that it is easier for the students to follow than the other methods. I feel, however, that the way a body of knowledge was unraveled is not necessarily the best way to teach the subject to students. The topics tend to be fragmented and cannot take full advantage of the conciseness of vector calculus. Students are puzzled at, and often form a mental block to, the subsequent introduction of gradient, divergence, and curl operations. As a process for formulating an electromagnetic model, this approach lacks cohesiveness and elegance.

The axiomatic development usually begins with the set of four Maxwell's equations, either in differential or in integral form, as fundamental postulates. These are equations of considerable complexity and are difficult to master. They are likely to cause consternation and resistance in students who are hit with all of them at the beginning of a book. Alert students will wonder about the meaning of the field vectors and about the necessity and sufficiency of these general equations. At the initial stage students tend to be confused about the concepts of the electromagnetic model, and they are not yet comfortable with the associated mathematical manipulations. In any case, the general Maxwell's equations are soon simplified to apply to static fields,

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Equation (6–23) enables us to find the vector magnetic potential **A** from the volume current density **J**. The magnetic flux density **B** can then be obtained from $\nabla \times \mathbf{A}$ by differentiation, in a way similar to that of obtaining the static electric field **E** from $-\nabla V$.

Vector potential **A** relates to the magnetic flux Φ through a given area S that is bounded by contour C in a simple way:

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s}. \tag{6–24}$$

The SI unit for magnetic flux is weber (Wb), which is equivalent to tesla-square meter ($\mathrm{T \cdot m^2}$). Using Eq. (6–15) and Stokes's theorem, we have

$$\Phi = \int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{s} = \oint_C \mathbf{A} \cdot d\boldsymbol{\ell} \qquad \text{(Wb)}. \tag{6–25}$$

Thus, vector magnetic potential **A** does have physical significance in that its line integral around any closed path equals the total magnetic flux passing through the area enclosed by the path.

6–4 The Biot-Savart Law and Applications

In many applications we are interested in determining the magnetic field due to a current-carrying circuit. For a thin wire with cross-sectional area S, dv' equals $S\,d\ell'$, and the current flow is entirely along the wire. We have

$$\mathbf{J}\,dv' = JS\,d\boldsymbol{\ell}' = I\,d\boldsymbol{\ell}', \tag{6–26}$$

and Eq. (6–23) becomes

$$\boxed{\mathbf{A} = \frac{\mu_0 I}{4\pi} \oint_{C'} \frac{d\boldsymbol{\ell}'}{R} \qquad \text{(Wb/m)},} \tag{6–27}$$

where a circle has been put on the integral sign because the current I must flow in a closed path,† which is designated C'. The magnetic flux density is then

$$\begin{aligned} \mathbf{B} = \nabla \times \mathbf{A} &= \nabla \times \left[\frac{\mu_0 I}{4\pi} \oint_{C'} \frac{d\boldsymbol{\ell}'}{R}\right] \\ &= \frac{\mu_0 I}{4\pi} \oint_{C'} \nabla \times \left(\frac{d\boldsymbol{\ell}'}{R}\right). \end{aligned} \tag{6–28}$$

† We are now dealing with direct (non-time-varying) currents that give rise to steady magnetic fields. Circuits containing time-varying sources may send time-varying currents along an open wire and deposit charges at its ends. Antennas are examples.

It is very important to note in Eq. (6–28) that the *unprimed* curl operation implies differentiations with respect to the space coordinates of the *field point*, and that the integral operation is with respect to the *primed source coordinates*. The integrand in Eq. (6–28) can be expanded into two terms by using the following identity (see Problem P.2–37):

$$\nabla \times (f\mathbf{G}) = f\nabla \times \mathbf{G} + (\nabla f) \times \mathbf{G}. \tag{6–29}$$

We have, with $f = 1/R$ and $\mathbf{G} = d\boldsymbol{\ell}'$,

$$\mathbf{B} = \frac{\mu_0 I}{4\pi}\oint_{C'}\left[\frac{1}{R}\nabla \times d\boldsymbol{\ell}' + \left(\nabla\frac{1}{R}\right) \times d\boldsymbol{\ell}'\right]. \tag{6–30}$$

Now, since the unprimed and primed coordinates are independent, $\nabla \times d\boldsymbol{\ell}'$ equals 0, and the first term on the right side of Eq. (6–30) vanishes. The distance R is measured from $d\boldsymbol{\ell}'$ at (x', y', z') to the field point at (x, y, z). Thus we have

$$\begin{aligned}
\frac{1}{R} &= [(x-x')^2 + (y-y')^2 + (z-z')^2]^{-1/2};\\
\nabla\left(\frac{1}{R}\right) &= \mathbf{a}_x\frac{\partial}{\partial x}\left(\frac{1}{R}\right) + \mathbf{a}_y\frac{\partial}{\partial y}\left(\frac{1}{R}\right) + \mathbf{a}_z\frac{\partial}{\partial z}\left(\frac{1}{R}\right)\\
&= -\frac{\mathbf{a}_x(x-x') + \mathbf{a}_y(y-y') + \mathbf{a}_z(z-z')}{[(x-x')^2 + (y-y')^2 + (z-z')^2]^{3/2}}\\
&= -\frac{\mathbf{R}}{R^3} = -\mathbf{a}_R\frac{1}{R^2},
\end{aligned} \tag{6–31}$$

where $\mathbf{a}_R$ is the unit vector directed *from the source point to the field point*. Substituting Eq. (6–31) in Eq. (6–30), we get

$$\boxed{\mathbf{B} = \frac{\mu_0 I}{4\pi}\oint_{C'}\frac{d\boldsymbol{\ell}' \times \mathbf{a}_R}{R^2} \quad \text{(T)}.} \tag{6–32}$$

Equation (6–32) is known as ***Biot-Savart law***. It is a formula for determining **B** caused by a current I in a closed path C' and is obtained by taking the curl of **A** in Eq. (6–27). Sometimes it is convenient to write Eq. (6–32) in two steps:

$$\boxed{\mathbf{B} = \oint_{C'} d\mathbf{B} \quad \text{(T)},} \tag{6–33a}$$

with

$$\boxed{d\mathbf{B} = \frac{\mu_0 I}{4\pi}\left(\frac{d\boldsymbol{\ell}' \times \mathbf{a}_R}{R^2}\right) \quad \text{(T)},} \tag{6–33b}$$

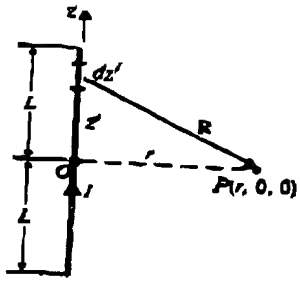

FIGURE 6-5
A current-carrying straight wire (Example 6-4).

which is the magnetic flux density due to a current element $I\,d\ell'$. An alternative and sometimes more convenient form for Eq. (6-33b) is

$$\boxed{d\mathbf{B} = \frac{\mu_0 I}{4\pi}\left(\frac{d\ell' \times \mathbf{R}}{R^3}\right) \quad \text{(T)}} \tag{6-33c}$$

Comparison of Eq. (6-32) with Eq. (6-10) will reveal that Biot-Savart law is, in general, more difficult to apply than Ampère's circuital law. However, Ampère's circuital law is not useful for determining **B** from I in a circuit if a closed path cannot be found over which **B** has a constant magnitude.

EXAMPLE 6-4 A direct current I flows in a straight wire of length $2L$. Find the magnetic flux density **B** at a point located at a distance r from the wire in the bisecting plane (a) by determining the vector magnetic potential **A** first, and (b) by applying Biot-Savart law.

Solution Currents exist only in closed circuits. Hence the wire in the present problem must be a part of a current-carrying loop with several straight sides. Since we do not know the rest of the circuit, Ampère's circuital law cannot be used to advantage. Refer to Fig. 6-5. The current-carrying line segment is aligned with the z-axis. A typical element on the wire is

$$d\ell' = \mathbf{a}_z\,dz'.$$

The cylindrical coordinates of the field point P are $(r, 0, 0)$.

a) *By finding* **B** *from* $\nabla \times \mathbf{A}$. Substituting $R = \sqrt{z'^2 + r^2}$ into Eq. (6-27), we have

$$\begin{aligned}\mathbf{A} &= \mathbf{a}_z \frac{\mu_0 I}{4\pi}\int_{-L}^{L} \frac{dz'}{\sqrt{z'^2 + r^2}} \\ &= \mathbf{a}_z \frac{\mu_0 I}{4\pi}\left[\ln\left(z' + \sqrt{z'^2 + r^2}\right)\right]\Big|_{-L}^{L} \\ &= \mathbf{a}_z \frac{\mu_0 I}{4\pi} \ln \frac{\sqrt{L^2 + r^2} + L}{\sqrt{L^2 + r^2} - L}.\end{aligned} \tag{6-34}$$

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